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Bayesian and Frequentist Modelling of West African Economic Growth: a Dynamic Panel Approach

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Abstract

The empirical outcomes of previous studies examining the relationship between economic growth and socio-economic indicators have been inconclusive and conflicting. To further probe into the study area, the current research employed a dynamic panel model estimated via three robust dynamic panel data estimators of the generalized method of moment (GMM), frequentist instrumental variable (IV) and the Bayesian IV on real and simulated data. Various model performance criteria such as Wald statistics, leave-out-one cross-validation and the Pareto *k* checks were used for validity verification. The results of the robust diagnostics checks and a model strength metric showed that the family of IV models outperformed the GMM. Thus, the estimation provided by the Bayesian IV is upheld and recommended in modelling dynamic panel data as it provides robust estimates of the parameters of interest.

Key words: dynamic panel data, economic growth, generalized method of moment, instrumental variable, socio-economic indicators.

1. Introduction

National economic development alludes to an expansion in the total efficiency of a nation or landmass. It is the amount more the economy produces than it did in the earlier period. To be exact, the correlation should eliminate the impacts of expansion (Becsi and Wang, 2002). Financial development is the advancement of Total national output (Gross domestic product) in the short, medium and long haul. It is the aftereffect of an expansion in esteem added delivered by every one of the organizations working inside a country. The increment in the worth added during a given period implies that the worldwide abundance of a country is rising and this shows itself in the development of per capita income and in a more significant level of prosperity.

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A wide scope of studies has explored the variables fundamental to economic development utilizing varying calculated and strategic perspectives, these investigations have set accentuation on an alternate arrangement of informative boundaries and offered different bits of knowledge to the wellsprings of economic development (Lensink and Morrissey, 2006). Venture is the most major determinant of economic development distinguished in the literature. The significance appended to speculation has prompted a colossal measure of experimental investigations analyzing the connection among venture and economic development (Artelaris *et al.,* 2007). It is additionally conceivable to accomplish total economic development without an expanded normal negligible efficiency yet through additional immigrants or higher rates of birth (Obadan, 2006).

Basu *et al.* (2005) noticed that Africa is the world's least fortunate continent. Various nations have as of late arose out of common conflicts that have seriously interfered with their formative endeavors while in different pieces of the continent, new outfitted struggles have erupted. These contentions and other antagonistic factors, outstandingly helpless climate conditions and crumbling as far as exchange, have prompted misfortune in monetary energy in the district in the course of the most recent twenty years. The authors recommend that what is required is a maintained and a considerable expansion in genuine per capita Gross domestic product development rates in these nations, combined with huge enhancements in friendly conditions. Endeavor to appraise the African mainland development is dependent on its Gross domestic product advancement and resident's buying equality. The monetary and social circumstance in sub-Saharan Africa accordingly stays delicate and defenseless against homegrown and outside shocks. Speculation stays curbed, restricting endeavors to broaden financial designs and lift development (Nkurunziza and Bates, 2004). This is in sharp difference to the happenings in the OECD nations where expansion in reserve funds and venture rate lead to economic development (Becsi and Wang, 2002).

Hu *et al.* (2014) suggest a generalized method of moment with individual specific fixed and threshold effects simultaneously. The issue of endogeneity in GMM was resolved by confirming that the symmetry conditions proposed by Arellano and Bond (1991) are legitimate. The proposed GMM estimator shows that the edge and incline boundary can be assessed precisely with consistency, and furthermore the finite sample dissemination of slant boundaries is well approximated by the asymptotic distribution (Blundell and Bond, 2000; Al-Sadoon et al., 2019).

Bardi et al. (2016) established empirically a positive and critical connection between structural policy and economic development utilizing a generalized moment method developed within dynamic panel structure. Sharma (2018) equally employs generalized method of moment estimator to re-examine wellbeing development relationship utilizing an unequal panel of 17 developed economies. The estimator takes care of endogeneity issues and through alternate model specifications it was established that population apply a positive and critical impact on both genuine income per capita just as development.

A few other authors who have written extensively on the estimation of economic growth both in Africa and globally as well as the practical application of GMM technique in modelling dynamic panel data are Lichtenberg (1992); Kiviet (1995); Blundell and Bond (1998); Agiomirgianakis *et al.* (2002); Ajayi (2003); Bengoa and Sanchez-Robles (2003); Agbeyegbe (2006); Obadan (2006); Lensink and Morrissey (2006); Dreher (2006); Levina (2011); Meraj (2013) and Adeboye *et al.* (2023). While GMM estimators depends strongly on the ratio of variance of the individual-specific effect and the variance of the general error term (see, e.g. Bun and Carree 2005), the IV largely depends on their individual specific effects that are uncorrelated with the explanatory variables x_{it}. A recent technique with limited approach in the literature is the Bayesian inference, which provides robust estimates of parameter of interest given because it involves updating the information based on prior statistics (Adesina and Obokoh, 2024). Limited studies have employed Bayesian statistics especially in recent times to estimate the parameters of interest in panel data. Some of the studies include Cho and Zheng (2021).

Dynamic panel estimation techniques were employed to establish the econometric bond between the selected macro-economic indicators of economic growth and purchasing power parity (PPP) across West African countries so that we can examine some desirable implications. Panel data has been established in the literature as all encompassing, in the areas of economic analysis [see the work of Adeboye and Agunbiade, 2019a and Adeboye and Agunbiade, 2019b]. Dynamic panel data estimation includes the work of Li et al. (2021) and Jin et al. (2021) who provided GMM estimation for dynamic panel models. The aim is to estimate the economic panel data with classical and Bayesian models using two-stage Least Square (2SLS) instrumental variable technique and compare with the GMM estimator proposed by Hu *et al.* (2014) to determine the approach that will provide the best estimates for dynamic panel. The adopted variables of measurement to validate the position of Basu *et al.* (2005) on the estimation of African continent growth are based on its GDP evolution and citizen's purchasing parity. The remaining part of the paper comprises Section 2, the material and methods, the results are presented in Section 3, and finally, Section 4 the provides conclusion.

2. Materials and Methods

The life data utilized were obtained mainly from UNESCO data site, which covers a period of 10 years ranging from 2008-2017 for selected West African countries as retrieved in the year 2018 while Monte Carlo simulation scheme was carried out using a data-generating procedure specified within a dynamic panel data model.

2.1. Model Specification

The relational model for this study is specified as

$$lGDP_{it} = \beta_{0it} + \beta_1(lPPP)_{1,it} + \beta_2(lGNI)_{2,it} + e_{it} \quad (1)$$

where i and t indicate the cross-sectional units (countries) and years under consideration respectively. GDP is the gross domestic product, PPP is the purchasing power parity and GNI is the gross national income of the West African countries while $\boldsymbol{e_{it}}$ is the unestimated residual. Considering the fact that the countries are diverse, a panel unit root test was carried out on the variables through the adoption of IPS (2003) test for individual unit root process given as

$$\Delta y_{it} = \rho_{it} y_{i,t-1} + \textstyle\sum_{L=1}^{p_i} \emptyset_{iL} \Delta y_{i,t-L} + z_{it}^{/} y + u_{it} \quad (2)$$

The LPS test is based on the assumption that the unit root can differ across the cross-sectional units in the model.

2.2. Estimation Methods

Two dynamic panel data estimation methods of the generalized method of moment and instrumental variable were employed. GMM was estimated according to the Arellano and Bond approach having fully taken care of endogeneity phenomenon and IV estimated via a 2sls technique.

2.2.1 Generalized Method of Moment (GMM)

Considering the first order model

$$y_{it} = X_{it}\beta + \delta y_{i,t-1} + \alpha_i + \varepsilon_{it} \quad (3)$$

and adopting the principle established by Hu *et al.* (2014), the first difference equation of (3) was observed to get rid of constant time of individual effects as

$$\Delta\, y_{it} = \Delta X_{it}(\beta)'\, \alpha_i + \Delta y_{i,t-1}(\delta)' + \Delta\varepsilon_{it} \quad (4)$$

$$y_{it} - y_{i,t-1} = \propto_i (x_{it} - x_{i,t-1})\beta' + (y_{i,t-1} - y_{i,t-2})\,\delta' + (\varepsilon_{it} - \varepsilon_{i,t-1}) \quad (5)$$

Considering that $\alpha_1 \neq \alpha_2$, it was established that the orthogonality conditions that exist between lagged values of y_{it} and the residual term ε_{it} are also valid in model (5). And without loss of generality, for any given t

$$X_{it}(\beta) = (x_{it}, 0)' or\ X_{it}(\beta) = (0.x_{it})' \quad (6)$$

And the first difference yields

$$\Delta X_{it}(\beta) = X_{i,t} - X_{i,t-1} \quad (7)$$

It should be noted that $\mathbf{X_{i,t-1}}$ satisfies the conditions of endogeneity and $\boldsymbol{\varepsilon_{it}'s}$ are serially uncorrelated. Thus, the orthogonality conditions are given by

$$E(X_{i,t-s}\, \Delta\varepsilon_{it}) = 0, \quad for\ s = 1, \cdots, t-1; t = 2, \cdots, T$$

Thus, within T observations in group i, Arellano and Bond (1995) suggested the fact that

$$\mathbf{E}\left[\begin{pmatrix} x_{1it} \\ x_{2it} \\ z_{1i} \\ \bar{x}_{1i} \end{pmatrix} (\boldsymbol{\eta}_{it} - \overline{\boldsymbol{\eta}}_{it})\right] = \mathbf{0} \quad \text{for some s} \neq \text{t}. \tag{8}$$

In principle, each valid instrument is extrinsic with respect to η_{it} subject to current, lagged, and future periods. Thus, there are a total of $[T(C1 + C2) + D1 + C1)]$ moment conditions for every observation.

$$\textbf{Let } \boldsymbol{W_i} = \begin{pmatrix} w'_{i1} \\ w'_{i2} \\ \vdots \\ w'_{iT} \end{pmatrix} \quad \text{and} \quad \boldsymbol{y_i} = \begin{pmatrix} y'_{i1} \\ y'_{i2} \\ \vdots \\ y'_{iT} \end{pmatrix} \tag{9}$$

W_i is assumed to be a $T\times$ $(1+ C1 + C2 + D1 + D2)$ matrix and $T+1$ observations available on y_i. Considering a matrix V_i consisting of $Ti - 1$ rows with instrument $\mathbf{v}'_{it}$ given as

$$\boldsymbol{V_i} = \begin{bmatrix} \boldsymbol{v}'_{i1} & \mathbf{0}' & \cdots & \mathbf{0}' \\ \vdots & \boldsymbol{v}'_{i1} & \ddots & \vdots \\ \mathbf{0}' & & \cdots & \boldsymbol{a}'_i \end{bmatrix} \tag{10}$$

Considering the transformation matrix, H, constructed as

$$\mathbf{H} = \begin{pmatrix} \boldsymbol{M^{01}} \\ \boldsymbol{T^{-1}i'T} \end{pmatrix} \tag{11}$$

where M^{01} denotes the first $T-1$ rows of the matrix (M^0) that creates deviations from group means. Thus, H replaces the last row of M^0 with a row of T^{-1}.

Let the $T \times 1$ column vector of disturbances be represented as

$$\eta_i = [\eta_{i1}, \eta_{i2}, \ldots, \eta_{iT}] = [(\varepsilon_{i1} + u_i), (\varepsilon_{i2} + u_i), \ldots, (\varepsilon_{iT} + u_i)]', \tag{12}$$

then

$$\boldsymbol{H_\eta} = \begin{pmatrix} \eta_{i1} - \bar{\eta}_i \\ \vdots \\ \eta_{i,T-1}\bar{\eta}_i \\ \bar{\eta}_i \end{pmatrix}$$

$$E[V' H_{\eta i}] = E[g_i] = 0. \tag{13}$$

The moment condition that follows from (13) is given as

$$plim\, n^{-1} \sum_{i=1}^{n} V_i' H_{\eta_i} \tag{14}$$

Explicitly given as

$$plim\ n^{-1} \sum_{i=1}^{n} V_i' H \begin{bmatrix} y_{i1} - \delta y_{i0} - x'_{1i1}\beta_1 - x'_{2i1}\beta_2 - z'_{1i}\alpha_1 - z'_{2i}\alpha_2 \\ y_{i2} - \delta y_{i1} - x'_{1i2}\beta_1 - x'_{2i2}\beta_2 - z'_{1i}\alpha_1 - z'_{2i}\alpha_2 \\ \vdots \\ y_{iT} - \delta y_{i,T-1} - x'_{1iT}\beta_1 - x'_{2iT}\beta_2 - z'_{1i}\alpha_1 - z'_{2i}\alpha_2 \end{bmatrix} \tag{15}$$

$$= plim\ n^{-1} \sum_{i=1}^{n} m_i = plim \bar{m} \tag{16}$$

Then the GMM estimator $\hat{\theta}$ is obtained by minimizing

$$q_{it} = \bar{m}^{/} A \bar{m} \tag{17}$$

The best weighting result of matrix A is derived as the inverse of the asymptotic covariance matrix of $\sqrt{n}\bar{m}$ and the solution to the minimizing problem of q_{it} with respect to the parameter vector $\boldsymbol{\theta}$ is the GMM estimator given as

$$\hat{\theta}_{GMM} = [(\textstyle\sum_{i=1}^{n} W_i^{/} H V_i)(\sum_{i=1}^{n} V_i^{/} H \eta_i \eta_i^{/} H^{/} V_i)^{-1}(V_i^{/} H^{/} W_i)]^{-1}$$
$$[(\textstyle\sum_{i=1}^{n} W_i^{/} H V_i)(\sum_{i=1}^{n} V_i^{/} H \eta_i \eta_i^{/} H^{/} V_i)^{-1}(V_i^{/} H^{/} W_i)] \tag{18}$$

2.2.2. Instrumental Variable (IV)

This is used to estimate causal relationships when controlled experiments are not feasible. Going by equation (3), the first order model becomes

$$y_{it} = x_{it}^{/} \boldsymbol{\beta} + z_i^{/} \boldsymbol{\alpha} + \varepsilon_{it}. \tag{19}$$

The underlying assumption of equation (19) clearly specifies that individual specific effects z_i are uncorrelated with the explanatory variables x_{it}. Thus, the model becomes

$$y_{it} = \mathbf{x}^{/}_{1it} \boldsymbol{\beta}_1 + \mathbf{x}^{/}_{2it} \boldsymbol{\beta}_2 + \mathbf{z}^{/}_{1i} \boldsymbol{\alpha} 1 + \mathbf{z}^{/}_{2i} \boldsymbol{\alpha}_2 + \varepsilon_{it} + u_i \tag{20}$$

where $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2)$ and $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2)$.

The strategy for estimation involved deviations of group means to have

$$y_{it} - \bar{y}_i = (\mathrm{x}_{1it} - \bar{x}_{1i})^{/} \beta_1 + (\mathrm{x}_{2it} - \bar{x}_{2i})^{/} \beta_2 + \varepsilon_{it} - \bar{\varepsilon}_i. \tag{21}$$

Representing the model variables as a weighted instrument given as

$$w_{it}^{/} = (x_{1it}^{/}, x_{2it}^{/}, z_{1i}^{/}, z_{2i}^{\prime}). \tag{22}$$

The transformed variables of equation (22) becomes

$$w_{it}^{*/} = w_{it}^{/} - \hat{\theta} \bar{w}_i^{/} \text{ and } y_{it}^{*} = y_{it}^{/} - \hat{\theta} \bar{y}_i^{/} \tag{23}$$

where $\hat{\theta}$ is a BLUE of θ. Thus, instrumental variables are given as

$$v_{it}^{/} = [(\mathrm{x}_{1it} - \bar{x}_{1i})^{/} + (\mathrm{x}_{2it} - \bar{x}_{2i})^{/} + z_{1t} - \bar{x}_{1t}] \tag{24}$$

And these are pile up in the rows of matrix $nTx(C1 + C2) + D1 + C1)$ denoted as **V**. The time-invariant variables and group means are repeated for the 3rd and 4th set of instruments, and the instrumental variable estimator becomes

$$(\beta\alpha)_{iv} = [(\mathrm{W}^{*/}\mathrm{V})(\mathrm{V}^{/}\mathrm{V})^{-1}(\mathrm{V}^{/}\mathrm{W}^{*})]^{-1}[(\mathrm{W}^{*/}\mathrm{V})(\mathrm{V}^{/}\mathrm{V})^{-1}(\mathrm{V}^{/}\mathrm{y}^{*})]. \tag{25}$$

The Bayesian alternative to the frequentist IV is presented in Section 2.3.

2.3. The Bayesian Implementation for the Instrumental Variable (IV)

Beyond the frequentist IV, the Bayesian IV was conducted based on multilevel approach. The Bayesian statistics involves the combination of likelihood and the prior distribution to obtain another distribution known as posterior distribution.

2.3.1. Prior Distributions and Sampling procedure

Prior distribution at group level assumed that parameters of interest come from a multivariate normal distribution having zero mean and unknown covariance matrix $\boldsymbol{\Sigma}$.

$$\epsilon \sim N(0, \boldsymbol{\Sigma}) \tag{26}$$

Covariances between group-level parameters are generally of different groupings factors and assumed to be zero. The model can be simplified to

$$\epsilon_i \sim N(0, \boldsymbol{\Sigma_i}) \tag{27}$$

where i indexes grouping factors. In cases where there are different levels with additional level indexed by j and the grouping factors are not dependent, Eq. (27) leads to:

$$\epsilon_{ij} \sim N\left(0, \mathbf{M_j}\right) \tag{28}$$

The model parameters will result from the covariance matrices $\mathbf{M_j}$, and No-U-Turn Sampler (NUTS) to sample $\mathbf{M_j}$ as recommended by Hoffman and Gelman (2014). The parameters of $\boldsymbol{M}_j$ are selected in terms of correlation matrix $\boldsymbol{\Omega_j}$ and a vector of standard deviations σ_j through

$$\boldsymbol{M}_j = \boldsymbol{D}(\sigma_j)\boldsymbol{\Omega_j}\boldsymbol{D}(\sigma_j) \tag{29}$$

2.3.2. The Sampling and Diagnostics Checks

The sampling method is the NUTS Sampler. NUTS is an extended Hamiltonian Monte-Carlo (HMC) which allows setting parameters and eliminates the need for hand-tuning Hoffman and Gelman (2014). Software package by R core team (2024) was used to fit the model with brms package by Bürkner (2017), which uses stan processor. Diagnostic plots for acceptance of NUTS plots were conducted, Adesina (2021) has the details of the procedure.

The study adopted the Leave-one-out cross-validation (LOO-CV) for the diagnostic tests. In Bayesian analysis, the data are repeatedly subdivided into a training set y_{train} and a holdout set $y_{holdout}$ with the objective of fitting y_{train} yielding a posterior distribution

$$p_{train}(\theta) = p_{train}(\theta | y_{train}) \tag{30}$$

The Bayesian LOO-CV estimate of out-of-sample predictive fit is

$$pd_{loocv} = \sum_{i=1}^{n} log\, p_{post(-i)}\,(y_i) \tag{31}$$

and estimated as

$$\sum_{i=1}^{n} log\left(\frac{1}{S}\sum_{s=1}^{S} log\, p\,(y_i|\theta^{is})\right) \tag{32}$$

To compare between two or more models the lowest LOO suggests better model fit.

The k Pareto also assesses the reliability and approximate convergence rate of the Pareto smoothed importance sampling (PSIS). It follows that if $k < 0.5$ ('good') then the central limit theorem holds. If $0.5 \leq k < 1$, ('ok') then the variance of the raw importance ratios is infinite, but the mean exists. On the other hand, if $k > 0.7$ ('bad'), unreasonable convergence rates are observed and unreliable Monte Carlo error estimates, and finally, if $k \geq 1$ ('very bad'), the variance and the mean of the raw importance ratios does not exist.

2.4 Monte Carlo Simulation Scheme

Monte Carlo simulations method was used to generate alternative data necessary for fitting and validation of the suitability of the proposed economic growth model. According to Hu *et al.* (2014), the data generating procedure (DGP) is given by

$$y_{it} = \delta y_{i,t-1} + x'_{1it}\beta_1 + x'_{2it}\beta_2 + z'_{1i}\alpha_1 + z'_{2i}\alpha_2 + \varepsilon_{it} + u_{it} \tag{33}$$

$for\ i = 1, \cdots, N\ and\ t = 1, \cdots, T\ where\ \varepsilon_{it} \sim i.i.d.N(0,1), \quad u_{it} \sim i.i.d.N(0,1),$ $\alpha_1 = \alpha_2 = 0. z_{it,}\ u_{it}, \varepsilon_{it}$ are mutually independent random variables.

The design of Monte Carlo simulations was carried out to further examine both the effectiveness and finite sample properties of different estimators of parameter α. The cross-sectional units are as small as 20 while $T = 10$ is the largest time dimension used in the study. A balanced panel data was first simulated and the data was made dynamic by the deletion of 2nd time period (time 4) for all individuals. It was assumed rho and alpha are 0, while the parameters used are uniformly distributed.

3. Results and Discussions

The results of both real life and simulated data are presented in the following tables:

Table 1: Results of Generalized method of Moments (GMM)

	Real Life Data	Simulated Data
Economic Growth Indicators	**GMM (One step) Parameter Estimate**	**GMM (One step) Parameter Estimate**
l_GDP(-1)	0.936024 (0.0001)	0.0727352 (0.9891)
Constant	–0.00519 (0.9951)	-0.0088614 (0.8455)
l_PPP	0.32395 (0.1575)	-0.0237805 (0.7934)
l_GNI	0.04064 (0.4341)	0.0016380 (0.9269)

Table 2: GMM Model Diagnostic

	Real Life Data	Simulated Data
Test	**Estimates**	**Estimates**
Test for AR(1) errors	-1.33091 (0.1832)	-0.7451126 (0.4562)
Test for AR(2) errors	-0.47378 (0.6357)	0.1865387 (0.85202)
Sargan over-identification test	87.3077 (0.0001)	14.40015 (1.0000)
Wald (joint) test	6298.11 (0.0000)	0.1324148 (0.9979)

Table 3: Results of Instrumental Variable

Real Life Data		Simulated Data	
Indicators	**2SLS Estimates**	**Indicators**	**2SLS Estimates**
Constant	13.1335 (4.42e-112)***	Constant	12.47 (2e-16)***
PPP	-3.63842 (0.0238)**	(gdp,1)	0.000001408 (2e-16)***
GNI	5.6721 (7.36e-06)*	PPP	-0.00007405 (0.767)
		GNI	0.00003354 (0.550)

Table 4: IV Model Diagnostic

Real Life Data		Simulated Data
Test	**Estimates**	**Estimates**
F- statistic	219.195 (0.0000)	823.615 (2e-16)***
Wald (joint) test	18.5464 (0.0001)	226900 (2.2e-16)***
R^2	0.69433	0.9997

Note that the P-values are in parenthesis.

GMM and IV models specified from Tables 1 and 3 are given as

$$l_{GDP_{it}} = -0.00519 + \left(0.93602_{GDP(it-1)}\right) + 0.32395(l_{PPP})_{1,it} + 0.04064(l_{GNI})_{2,it} \quad (34)$$

$$l_{GDP_{it}} = -0.00886 + \left(0.0727_{GDP(it-1)}\right) - 0.0237(l_{PPP})_{1,it} + 0.0406423(l\_GNI)_{2,it} \quad (35)$$

$$GDP_{it} = 13.1335 - 3{,}63842(PPP)_{1,it} + 5.6721(GNI)_{2,it} \quad (36)$$

$$GDP_{it} = 12.47 + 0.000001408(GDP_{-1}) - 0.00007405(PPP)_{1,it} + 0.00003354(GNI)_{2,it} \quad (37)$$

Models (34) – (37) represent the empirical growth models estimated from both real life and simulated data. It is pertinent to note that models from the GMM technique give negative projections of African economic growth at constant values of the predictors, despite the absence of exogeneity while that of IV give positive projections with a more superior significant values as presented in Tables 2 and 4. Thus, the model in which its explanatory variables are more significant with improved validity checks is that of the instrumental variable.

The validity checks further revealed the absence of serial correlation among the variables due to the results of AR(1) and AR(2) while the Sargan test validates the instrumental variables. Similarly, the results reported in Table 3 shows that this instrument can be considered as exogenous given that the null hypothesis is not rejected at both 1% and 5% percent level, as posited by Bascle (2008). The other two macroeconomic instruments were individually and simultaneously tested for exogeneity to increase our confidence that both instruments can be considered as exogenous in this setting. Table 5 contains the estimates based on Bayesian Multilevel IV model.

Table 5: Bayesian Multilevel IV model

Specification	Estimate	Est.Error	l-95%CI	u-95%CI	$\hat{R}$	Bulk ESS	Tail ESS
sd(GDP_Intercept)	181914.25	109221.3	5516.60	409371.7	1.01	126	185
sd(GDP_GNI)	0.00	0.00	0.00	0.00	1.00	231	440
sd(GDP_PPP)	316677.86	214973.8	7218.73	66169.88	1.01	109	154
sd(logGDP_Int.)	2.60	0.48	1.83	3.71	1.00	282	343
sd(logGDP_PPP)	1.19	0.55	0.21	2.41	1.00	387	412
sd(logGDP_GNI)	0.00	0.00	0.00	0.00	1.00	262	306
cor(GDP_Int, GDP_GNI)	0.02	0.47	-0.83	0.87	1.02	85	221
cor(GDP_Int, GDP_PPP)	0.05	0.49	-0.83	0.88	1.00	239	457
cor(GDP_GNI, GDP_PPP)	-0.02	0.48	-0.88	0.80	1.00	446	473
cor(logGDP_Int, logGDP_PPP)	-0.57	0.28	-0.95	0.08	1.00	534	383
cor(logGDP_Int, logGDP_GNI)	0.31	0.51	-0.78	0.95	1.00	175	332
cor(logGDP_PPP, logGDP_GNI)	-0.04	0.49	-0.86	0.86	1.00	253	425

NB: Int-Intercept.

There are two models in Table 5, the standard deviation estimate model, and the correlation model. The estimates are provided in the second column. The estimation error in the third column, the upper and lower 96% confidence interval in the fourth and fifth column. The Rhat ($\hat{R}$) in the sixth column which serves as potential scale reduction factor on split chains. The Bulk ESS, and Tail ESS in the seventh and eight column. The Bulk ESS is a diagnostic test to determine sampling efficiency while Tail ESS is used to determine the sampling efficiency in the tails of the posterior respectively.

The Bayesian multilevel IV model based on Table 5 can be expressed in terms of random intercepts and random slope, correlations between predictors and Bayesian priors as given in equations (38) and (39) below, which represent the models with standard deviation and correlation respectively:

$$GDP_{it} = 181914.25 + 0.000(GPD\_GNI)_{(it)} + 316677.86(GDP\_PPP)_{it} + 2.60(GDP\_Intrecept)_{it} + 1.19(logGDP\_PPP)_{it} + 0.000(logGPD\_GNI)_{(it)} \quad (38)$$

$$GDP_{it} = 0.02(GPD\_Int)(GDP\_GNI)_{(it)} + 0.05(GPD\_Int)(GDP\_PPP)_{(it)} - 0.02(GPD\_GNI)(GDP\_PPP)_{(it)} - 0.57(logGDP\_Int)(logGDP\_PPP)_{it} + 0.31(logGPD\_Int)(logGDP\_GNI)_{it} - 0.04(logGDP\_PPP)(logGDP\_GNI)_{it} \quad (39)$$

The models predicted GDP using several predictors (GDP_GNI, GDP_PPP, log$GPD_GNI, logGPD_GNI$ and $logGPD\_Int$). The random intercept estimated in equation (38) accounts for the variation across units and it is assumed to follow a normal distribution with a standard deviation of 181914.25, which indicates substantial variation in GDP across countries. The zero variation between GDP and GNI implies that the countries' GNI has impacted favorably on the GDP without any variation while that of PPP suggests a large variation in the GDP of countries as occasioned by the countries' PPP. According to the correlation estimates provided in equation (39), the predictors are correlated with each other at different degrees, with correlations close to zero, suggesting little association between variables.

The estimated standard errors of the model as contained in the column 3 of Table 5 are negligible except for that of the interaction between GDP and PPP. This implies that the estimates of all other predictors are reliable with negligible uncertainty in their estimation. This was supported with credible intervals provided for all the predictors, with intervals which include zero except for that of PPP mentioned earlier. All the $\hat{R}$ are greater than 1.00 in all the cases indicating a good convergence for the Markov Chain Monte Carlo (MCMC) chains, with high values of ESS (ESS > 100), which suggests that the estimates are reliable and that the posterior distribution has been adequately sampled. This opinion is in tune with the work of Bürkner (2017) and Jiménez et al. (2022).

Table 6 contains the estimates of the response variables based on the intercept.

Table 6: Comparison of Intercept Estimates

Specification	Estimate	Est. Error	l-95% CI	u-95% CI	$\hat{R}$	Bulk ESS	Tail ESS
GDP_Intercept	4702.15	3421.77	-2638.15	10147.91	1.06	21	59
logGDP_Intercept	11.86	0.64	10.53	13.10	1.01	94	226

From Table 6, the regression estimate of the $(GDP)_{int}$ = 4702.15, zero included, while $(logGDP)_{int}$ = 11.86 is significant. The Rhat is close to 1 in both cases, which shows that the chain converged. The Bulk ESS and Tail ESS of $(GDP)_{int}$ are less than 100, whereas Tail ESS of $(log\ GDP)_{int}$ is greater than 100 showing that there is efficient sampling in the tails of the posterior distribution.

Draws were sampled using sampling (NUTS). For each parameter, Bulk ESS and Tail ESS are effective sample size measures, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat = 1). Table 7 shows the posetrior summary.

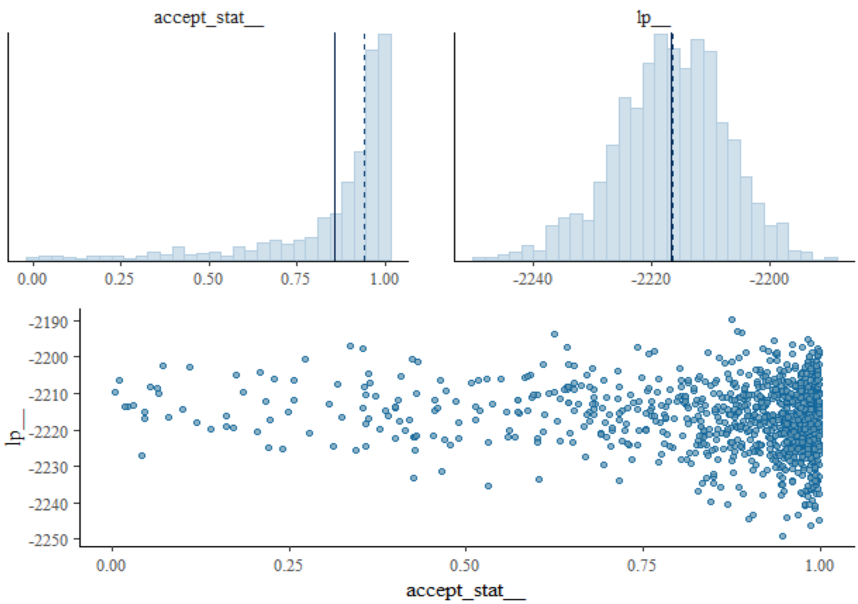

Figure 1: Acceptance diagnostic checks for NUT Sampler

Figure 1 shows that the acceptance probability of the sampler is nearly 100%, which shows the efficiency of the sampler for the model. The density on the upper right in Figure 1 shows that the data are well distributed. The accept_stat_ in Figure 1 shows that the sample cluster around 1.00 and majority close to 1.00 showing a high acceptance rate.

Table 7: Posterior Distribution Estimates

Specification	Estimate	Est.Error	Q2.5	Q97.5
b_GDP_Intercept	4.7021e+03	3.4217e+03	-2.6381e+03	1.0147e+04
b_logGDP_Intercept	1.1860e+01	6.4158e-01	1.0531e+01	1.3098e+01
sd_ID__GDP_Intercept	1.8191e+05	1.0922e+05	5.5165e+03	4.0937e+05
sd_ID__GDP_GNI	1.7202e-07	5.0712e-08	9.3678e-08	2.8880e-07
sd_ID__GDP_PPP	3.1667e+05	2.1497e+05	7.2187e+03	7.6616e+05
sd_ID__logGDP_Intercept	2.6008e+00	4.7777e-01	1.8265e+00	3.7144e+00

Table 7: Posterior Distribution Estimates (cont.)

Specification	Estimate	Est.Error	Q2.5	Q97.5
sd_ID__logGDP_PPP	1.1864e+00	5.5401e-01	2.1320e-01	2.4061e+00
sd_ID__logGDP_GNI	1.0017e-13	6.7751e-14	3.2062e-14	2.8442e-13
cor_ID__GDP_Intercept __GDP_GNI	2.1594e-02	4.7442e-01	-8.2537e-01	8.7270e-01
cor_ID__GDP_Intercept __GDP_PPP	4.5414e-02	4.9268e-01	-8.2671e-01	8.8462e-01
cor_ID__GDP_GNI __GDP_PPP	-2.2093e-02	4.7607e-01	-8.8198e-01	8.0275e-01
cor_ID__logGDP_Intercept __logGDP_PPP	-5.6823e-01	2.8381e-01	-9.4641e-01	7.9702e-02
cor_ID__logGDP_Intercept __logGDP_GNI	3.0527e-01	5.0828e-01	-7.8195e-01	9.4885e-01
cor_ID__logGDP_PPP __logGDP_GNI	-4.3449e-02	4.8786e-01	-8.5531e-01	8.6349e-01
sigma_GDP	2.7149e+05	1.6926e+04	2.4059e+05	3.0696e+05
sigma_logGDP	3.3815e-01	2.2685e-02	2.9746e-01	3.8724e-01
Intercept_GDP	4.7021e+03	3.4217e+03	-2.6381e+03	1.0147e+04
Intercept_logGDP	1.1860e+01	6.4158e-01	1.0531e+01	1.3098e+01
rescor__GDP__logGDP	7.0127e-01	4.7306e-02	5.9782e-01	7.8346e-01

Table 7 shows the posterior summary, which shows the model is similar to that of the estimates in Table 5. It was computed from 1000 by 143 log-likelihood matrix. The models for the standard deviation and the correlation estimates can as well be specified in the neighborhood of model (38) and (39). Table 8 and Table 9 contain the leave-out-one cross validation estimates and Pareto k diagnostic tests.

Table 8: LOO

Specification	Estimate	SE
elpd_loo	-2031.1	76.6
p_loo	69.5	34.4
Looic	4062.3	153.3

The elpd_loo (-2031.1) is the Bayesian leave-one-out (LOO) estimate of the expected log pointwise predictive density (ELPD), it can either be positive or negative. Large ELPD values indicate good estimated predictive performance, when comparing models, a larger ELPD suggests a better predictive performance. The p_loo is the difference between elpd_loo and the non-cross-validated log posterior predictive density. If p_loo < the number of parameters p, then the model is likely to be misspecified. The p_loo is 69.5 greater than the number of parameters.

Table 9: Pareto k diagnostic values

Specification	Count	Pct.	Min. ESS
(-Inf, 0.67] (good)	139	97.2%	71
(0.67, 1] (bad)	1	0.7%	-
(1, Inf) (very bad)	3	2.1%	-

Table 9 shows that out of 143 data points, 139 (97.2%) fall under good samples, 1 (0.7%) falls under bad sample, while 3 (2.1%) fall under very bad samples. The model proved to be a very good one.

Table 10 shows the R^2 statistics for both Bayes and (leave-out-one) LOO.

Table 10: Measure of Determination for Bayes and LOO Estimates

Specification	Estimate	Est.Error	Q2.5	Q97.5
	Baye_R2			
R2GDP	0.9174490	0.005425372	0.9055567	0.9261627
R2logGDP	0.9796951	0.001253107	0.9770527	0.9819441
	LOO_R2			
R2GDP	0.9150547	0.05972399	0.7529397	0.9818688
R2logGDP	0.9758178	0.01030771	0.9491834	0.9894607

The R^2 for both Bayes and LOO are very high with the R^2 values of 0.9174490 and 0.9150547 (91.75% and 91.51%) respectively, higher than that of frequentist IV model (0.69433). The two tail 95% confidence interval Q2.5 and Q97.5 shows that both R^2 are significant since the interval does not include zero.

4. Conclusions

Instances of African economic development have become the concern of many international agencies and governments at various levels, hence the needs for its continuous evaluation. Moreover, the opinions posited by previous studies examining the relationship between economic growth and socio-economic indicators have been indecisive and conflicting due to different sample periods, variables used, countries studied and econometric techniques employed. Thus, dynamic panel data estimation techniques of the generalized method of moment and instrumental variables (both the classical and the Bayesian) were employed to revisit the estimation. GMM was estimated according to the Arellano and Bond approach having fully taken care of endogeneity phenomenon as established by Hu *et al.* (2014) and IV estimated via a two-stage least square (2SLS) technique; it was discovered that the instrumental variable technique outperformed GMM based on robustness of the estimated models and the adopted model selection criteria. The preferred technique works well for both life and simulated data and Monte Carlo simulations reveal that the two methods have very good finite sample performance and give a positive projection of African economic growth compared to GMM, which gives a negative projection with weak validity criteria.

It is pertinent to emphasize the robustness of the adopted Bayesian IV in providing more reliable policy insights in terms of its consistency in handling endogeneity issues in data-driven approaches, which can improve the accuracy of policy recommendations. As established with the fitted IV models, policy makers should prioritize the growth of the countries national income and create more leverage in the purchasing power parity of citizenries to enhance sustainable economic development.

It is pertinent to note that the greater focus of this current research is in the area of opinionating a robust estimation technique for a dynamic panel model through the modeling of African economic growth, and this has been vigorously established in the Bayesian IV. This technique, however, is recommended for the expedition of current economic data with more diverse econometric variables for a more robust contribution to the field of econometrics, as it concerns the modeling of economic growth.

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